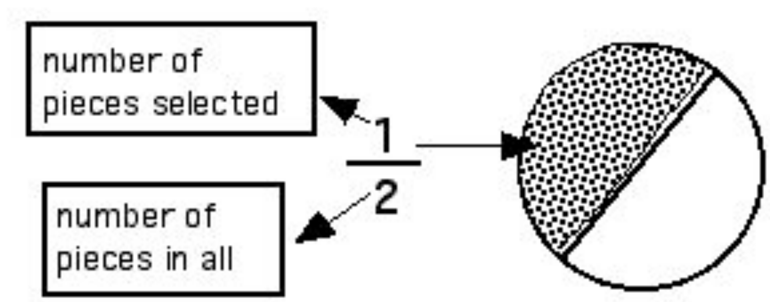
Five meanings of fraction

There are five meanings of fraction that are relevant to middle years mathematics. The first three meanings should have been developed in the early years (although the measure meaning may have only been developed by some teachers). The fourth and fifth meanings need to be developed in the middle years. Please note that the order of listing does not imply an order of teaching.

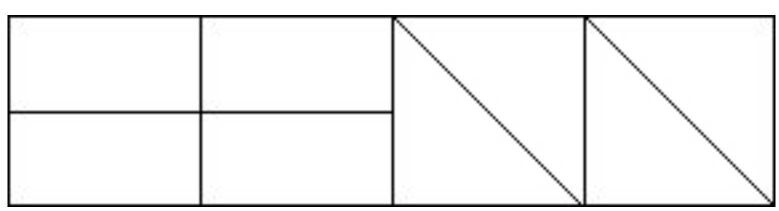
1. The cut meaning (also known as the part of a whole meaning)

When a 5-year old child says "I ate half of the cookie." he/she is expressing a part-whole relationship. The child uses 'half' not in the sense of a number but in the sense of an actual or imagined action that involves cutting a whole physical object in the middle. The imagined or actual action of cutting a whole object into 'n' equivalent/equal parts underlies the cut meaning of fraction. We represent each part symbolically by the fraction notation '1/n'. The circle diagram here indicates this.



'Equivalent/equal' means equivalent/equal according to length, area, or volume. In the case above, the two pieces of the circle are equal in area and they happen to look the same. That does not need to be the case.

Consider a granola bar cut in the way shown in the diagram. The 8 pieces do not all look the same. Yet each piece is 1/8 of the granola bar because the pieces have the same area.



In summary, the cut meaning of fraction involves cutting a naturally existing whole into equal parts according to measurable qualities such as length, area, volume, mass, etc. An example for length could be a string cut into 4 parts of equal length. Each part is 1/4 of the length of the whole string. An example for area is the granola bar or circle example above. An example for volume could be a loaf of bread cut into 8 parts having the same volume (a difficult thing to actually do). Each part is 1/8 of the volume of the whole loaf.

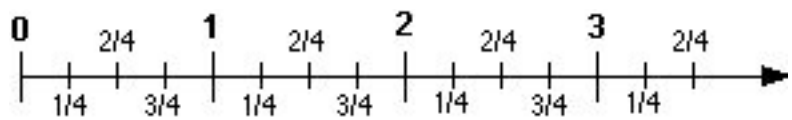
2. The part of a group or set meaning

The part of a group meaning does not involve cutting a whole into equal parts. Nor does it involve a natural whole. The part of a group meaning involves selecting objects from a group on some basis. A group is not a natural whole as is a pie (for example). Unlike the cut meaning, the part of a group meaning does not require that the objects be of the same size or type.

For example, suppose there are 23 books of varying size and content on a shelf where 14 of them are novels. We can represent this situation by the fraction 14/23. In this case, we mean by 14/23 that 14 out of the 23 books are novels. This meaning is significantly different from the 'part of the whole' meaning. The part of a group/set meaning involves mentally placing discrete things into categories (e. g. red, prime), a different enterprise than cutting up things according to length, area, or volume, etc.

3. The name for a point (or measure) meaning

The name for a point meaning involves associating marks on measuring devices such as rulers with fraction names. The cut meaning underlies this as the marks are obtained by cutting a section of a line into equal parts. Notice in the number line here that there are a lot of 1/4 lengths (for example). Yet each is different because they each name a different point on the number line (e. g. 1/4, 1 1/4, 2 1/4, etc.).



The name for a point meaning is the most abstract of the first three meanings of fraction, and, overall, the most useful for purposes of teaching computational skills.

A number line that contains fraction-named points can serve as a ruler for measuring length using fractional amounts of units of length. For this reason, some refer to the name for a point meaning as the measure meaning of fraction.

4. Ratio meaning

A fraction can be used to name a ratio. For example, 3/4 can mean a ratio of 3 to 4. This has nothing to do with wholes, groups, or name for a point. It involves making a comparison between two quantities.

Ratio is a statement about a numerical relationship (specifically, a comparison) between two quantities that may or may not involve different kinds of stuff. Suppose the ratio between flour and butter in a recipe is 5 cups flour to 3 tablespoons butter (5/3). In this situation, the relationship is between the same kind of stuff, volume, but it does involve different units. Suppose that in an eco-system, the optimal ratio between deer and forage is 1 deer to 2 tons of forage per square mile (1/2). This ratio involves different kinds of stuff, a count of discrete entities (deer) and a measurement of weight and area. Ratio may be explicit as in the examples above or it may be implicit. If I have two-thirds (2/3) as many notebooks as Harry, this implies a ratio of 2 to 3 (I have 2 notebooks for each 3 that Harry has).

We can name ratios using a variety of notations (e. g. 5 to 3, 5 : 3, 5/3). Fraction notation has an advantage when working with ratios. For example, we can equate two ratios when we are solving problems about similar triangles (e. g. x/5 = 7/10). Fraction notation leads to relatively simple methods for solving such equations.

5. Indicated division meaning

The indicated division meaning has nothing to do with wholes, groups, or measurement. It refers to the fact that the fraction bar, '/', can be a way of saying to divide. One application of this meaning is converting fraction notation to decimal notation, in which we divide the numerator of a fraction by its denominator. For example, 1/2 is 1 ÷ 2 or .5. For example, 3/4 is 3 ÷ 4 or .75.

Some of the confusion about ‘of’ and ‘x’ when working with fractions can be attributed to a lack of understanding that the symbol, ‘/’ can be used to indicate ‘divide’. In other words, there is another notational agreement for divide besides the symbol ‘÷’.